CHAPTER 8

Pick a Number

Revisiting Open Sentences

OVERVIEW

This lesson gives students experience with solving equations. Students each pick a number between zero and twenty-five and use it with the following directions: Multiply the number by two and then add seven. Students present their result, and the others figure out what their starting number was and explain their strategies. Students later write open sentences to represent the problems. For extensions, they repeat the experience for other directions and then make up their own problems.

BACKGROUND

Before experiencing this lesson, students should have had experience exploring true, false, and open sentences. (See Chapter 2.) In this lesson, students first figure out answers to number problems presented only with words. Using problems that come from students' own results keeps interest high and encourages students to stay engaged.

When following the directions to multiply a number by two and then add seven, students arrive at different results depending on what number they started with. A result of fifty-seven, for example, means that the starting number was twenty-five. Two methods typically emerge from the students when they figure this out. One is to guess and check. For example, they might guess that the starting number is twenty and then check it by doing $(20 \times 2) + 7$. This results in forty-seven. Because forty-seven is less than fifty-seven, this guess is too small. The lesson explains how to use a number line to keep track of guesses that are too big or too small.

Another method that typically comes up in a class is the "undoing" method. It seems natural for students first to "undo" the addition. They either subtract by thinking, "Fifty-seven minus seven," or they add by thinking, "What number do I need to add to seven to get fifty-seven?" Then they figure out, either by multiplying or dividing, that the number that was multiplied by two to get fifty is twenty-five.

For this problem, all results are odd numbers. That's because no matter what number you start with, multiplying it by two will produce an even number. Then adding on seven will result in an odd number because the

sum of an odd number and an even number is always odd. If a student comes up with an even result, and hasn't made a computation error, then the starting number involved a fraction. This situation occurred in the vignette that follows.

VOCABULARY

equation, open sentence

MATERIALS

- none

TIME

- one period to introduce, plus additional time for extensions

The Lesson

To begin the lesson, I said to the students, "Think of a number between zero and twenty-five. Don't tell me or anyone else what it is!" I paused for a moment and then asked, "Does everyone have a number?" The students nodded.

I continued, "Now listen carefully and follow my directions. Take your number and multiply it by two to get a new number." I paused to give the students time to think.

"Now add seven to your new number," I said. While the students were calculating in their heads, I wrote on the board:

Number multiplied by two, plus seven, results in _____

I wrote this on the board to help students realize that words, like numbers, can represent mathematical ideas. Also, it gave students a reference for what they were supposed to do. By the time I finished writing on the board, most of the students had a hand up, indicating they were ready to share. I called on Nick. "What result did you get?" I asked him.

"Fifty-seven," Nick replied.

"Did anyone else get fifty-seven?" I asked. No one else did. If other students had, I would have asked them to remain quiet for the following part of the discussion.

I then said, "Try to figure out what number Nick started with." I paused for a few moments to give students time to think. When about half of the students had raised a hand, I called on Joshua.

"Nick started out with twenty-five," Joshua said.

"That's right," Nick said.

"Let's check it and see how Nick could get fifty-seven if he picked the number twenty-five," I said. "What did I ask you to do first?"

"Pick a number," Tina said.

"Then what?" I continued.

"You multiply it by two," Jazmin replied.

"The last thing you said to do was to add seven," Cameron added.

As the students told me what we had done, I wrote the following on the board as a way to check Joshua's guess of twenty-five as Nick's starting number.

$(25 \times 2) + 7 = 57$
$50 + 7 = 57$

"It works!" I said. "How did you figure it out, Joshua?"

"I subtracted seven from fifty-seven to get fifty. Twenty-five is half of fifty," Joshua explained.

"Why did you subtract seven?" I asked Joshua. Joshua's method made sense to me, but I questioned him anyway for three reasons. One reason was to learn more about how Joshua was thinking; I've learned that correct answers sometimes mask misconceptions. Second, it's important in classroom discussions to question students not only when they're wrong but also when their thinking is solid; in this way, explaining your thinking becomes a regular part of class discussions. And third, I suspected that not all of the other students followed or understood how Joshua reasoned, and having him explain would slow down the discussion and, perhaps, help others catch up.

Joshua answered, "Well, you have something plus seven is fifty-seven, so the something must be fifty. Fifty-seven minus the seven is fifty."

"So you were going backward from the fifty-seven and undid the addition by subtracting?" I asked. Joshua nodded. "And how did you get to twenty-five?"

"Kind of the same way," Joshua responded. "Something times two was fifty, so the something had to be twenty-five."

I turned to the class. "Show thumbs up if you understand how Joshua reasoned." More than half of the students showed a thumb up. The others needed experience with more examples.

"So twenty-five is the number you started with, Nick," I confirmed. He nodded. On the board, I recorded Nick's result under the instructions I had written:

Number multiplied by two, plus seven, results in _____
Nick $(25 \times 2) + 7 = 57$

"Who came up with a different result?" I then asked. I called on Carmen.

"Forty-three," she said.

"Did anyone else get forty-three?" I asked the students. No one had.

"Figure out what number Carmen started with, and raise your hand when you think you know," I said to the class. I gave the students time to think and then called on Keith.

"I think that Carmen's starting number was eighteen," Keith said.

"Is that right, Carmen?" I asked. She said yes.

"How did you figure it out?" I asked Keith.

Keith explained, "I did it backwards, kind of like Joshua, but I divided at the end instead of multiplying. I started with forty-three and

then I subtracted seven, and that got me to . . . oh yeah, that got me to thirty-six. Then I divided thirty-six by two to get eighteen."

"Did someone do it differently?" I asked.

Cameron said, "First I subtracted, forty-three minus seven. What I did was forty-three minus three gets to forty, and then I have to subtract four more and I get down to thirty-six. Actually, from there I did it the same way as Keith. Well, I didn't exactly. I didn't divide. I multiplied."

"What did you multiply?" I asked Cameron.

She explained, "When I had thirty-six, I asked myself, 'What number times two gives thirty-six?' Then I thought of eighteen."

"I think that your way is sort of different than Keith's strategy, but related because multiplication and division are related," I said to Cameron.

I recorded Carmen's result below Nick's.

Number multiplied by two, plus seven, results in ____
Nick $(25 \times 2) + 7 = 57$
Carmen $(18 \times 2) + 7 = 43$

"Can I share my result?" Steve asked. I nodded. He said, "I got eleven."

I said to the class, "Think about Steve's result and raise your hand when you know the number he started with," I said. I waited until about three-fourths of the students had their hands raised, then called on Kiko.

"Steve started with two," Kiko said.

"How did you figure it out?" I asked.

"First I did eleven minus seven, which equals four. Then I divided four by two, which equals two," Kiko explained.

"You used the same strategy as Joshua and Keith," I commented. Kiko nodded.

"Was two the number you started with, Steve?" I asked. Steve indicated it was.

"Did anyone use a different method to figure out Steve's number?" I asked.

Louise said, "I just guessed. First I guessed three and it was too big. Then I guessed one and it was too small. So it had to be two, and it worked."

"What happened when you guessed three?" I asked.

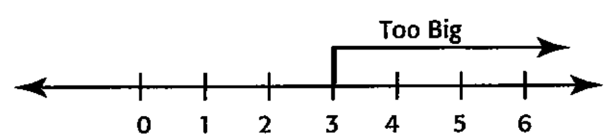
Louise said, "I did three times two is six, and six plus seven is thirteen." I wrote on the board:

$(3 \times 2) + 7 = 13$

"How did you know that three was too big?" I asked.

"Steve's number was eleven, and I got thirteen," Louise answered.

I said, "When you guess, as Louise did, it helps some people to keep track of what happens on a number line." I drew a number line on the board to show the students what I meant:



"Would four be a good next guess for Louise?" I asked the class. Hands shot up. I called on Tina.

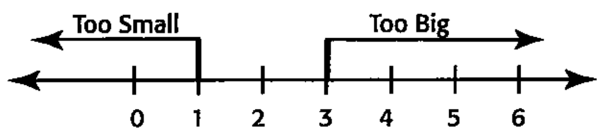
She said, "If three is too big, then four would be really too big."

I pointed to 4 on the number line and said, "Yes, it's definitely in the Too Big section." I turned back to Louise. "So then you guessed one?"

She said, "I went down from three to one, but I went too far. I did one times two is two plus seven is nine." I recorded on the board:

$(1 \times 2) + 7 = 9$

"Nine was too small," Louise concluded. "So I did two and it was just right. It's in the middle and it worked." I marked the number line to show that one was too small a guess.



I added Steve's result to the list on the board:

Number multiplied by two, plus seven, results in _____

Nick	$(25 \times 2) + 7 = 57$
Carmen	$(18 \times 2) + 7 = 43$
Steve	$(2 \times 2) + 7 = 11$

Lucy shared her result next. "I got thirty-seven as my result," she said. Several students raised their hands, indicating that they also had gotten Lucy's result. I asked these students to listen as the others tried to figure out the starting number for thirty-seven.

I said to the class, "Let's try Louise's method to find out what number Lucy started with. Who would like to guess a number to test?"

"It has to be less than eighteen," Cameron said, looking at what I had recorded on the board. "I guess fifteen." I wrote on the board:

$(15 \times 2) + 7$

Terry did the figuring. He said, "Fifteen times two is thirty, and thirty plus seven is thirty-seven. It works!"

"You guessed right, Cameron," I said. "Who can figure it out using Joshua and Keith's strategy?"

Lisa said, "You do it backwards. They would take seven from thirty-seven, which gives thirty. Then you do thirty divided by two, which is the same as finding half. That's undoing the multiplying by two. You get fifteen."

I said to the class, "Show thumbs up if you understand how Lisa reasoned." More students showed a thumb up than when I had asked about how Joshua reasoned, but a few still weren't sure. I recorded Lucy's result:

Number multiplied by two, plus seven, results in _____

Nick	$(25 \times 2) + 7 = 57$
Carmen	$(18 \times 2) + 7 = 43$
Steve	$(2 \times 2) + 7 = 11$
Lucy	$(15 \times 2) + 7 = 37$

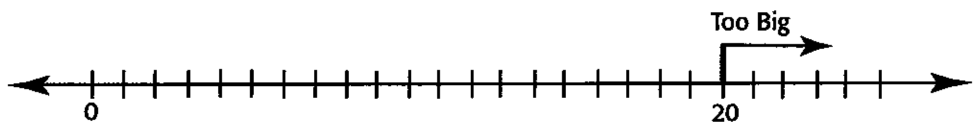
"Let's try one more different result," I said. Joshua looked at me hopefully. He had been raising his hand eagerly throughout the activity, and I now called on him.

"I got thirty-two," Joshua said. No one else did. The sly smile on Joshua's face gave me the clue to think for a moment about his result. All of the other results so far had been odd. That made sense because they all multiplied their initial number by two, which gives an even number, and then they added seven, an odd number. When you add an odd number to an even number, the answer will always be odd.

I asked the class, "Would a good guess for Joshua's starting number be twenty?"

"No," several students quickly replied.

"It's too big!" Rayno said. "Fifteen gave thirty-seven, so this has to be smaller." I drew a number line and indicated on it that all numbers from 20 on up were too big:

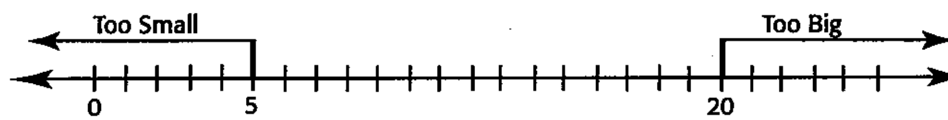


"How about five?" I asked.

"I think that would be too small," Cameron said.

"Are you sure?" I asked.

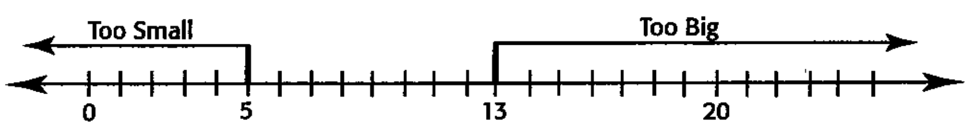
Cameron answered, "Well, five times two is ten, and plus seven is only seventeen. It's way too small." I indicated this on the number line:



"How about thirteen?" Jazmin suggested. I wrote on the board:

$(13 \times 2) + 7$

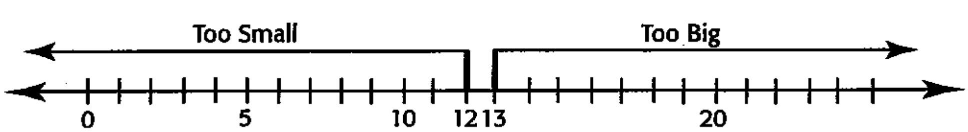
Jazmin figured, "Thirteen times two is twenty-six, plus seven is . . . thirty-three. It's a little too big." I extended the Too Big arrow on the number line so it started from 13:



"What about twelve?" Jeremy suggested. I wrote on the board:

$(12 \times 2) + 7$

Jeremy figured, "Twelve times two is twenty-four and seven is thirty-one. Hey, it's too small!" I adjusted the Too Small arrow on the number line:



The students thought for a few moments and then hands began to go up. I called on Louise. "It's twelve and a half!" Louise said. "Twelve and a half times two is twenty-five, and twenty-five plus seven is thirty-two."

"I didn't know you could use fractions," Kenny said.

"I hadn't thought about using fractions myself," I said, "but Joshua did." Rayno's hand was up.

"What are you thinking, Rayno?" I asked.

"Louise is right. If you do thirty-two minus seven, you get twenty-five. You can't divide twenty-five by two without leftovers. Twenty-four divided by two is twelve, plus one-half would be twelve and a half. Twelve and a half plus twelve and a half equals twenty-five," Rayno explained. Joshua agreed that his starting number had been twelve and a half.

Jeremy said, "You could see from the number line how to guess. Twelve is too small and thirteen is too big. Twelve and a half is a half away from twelve and a half away from thirteen."

I added Joshua's equation to the list:

Number multiplied by two, plus seven, results in _____

Nick	$(25 \times 2) + 7 = 57$
Carmen	$(18 \times 2) + 7 = 43$
Steve	$(2 \times 2) + 7 = 11$
Lucy	$(15 \times 2) + 7 = 37$
Joshua	$(12\frac{1}{2} \times 2) + 7 = 32$

Writing Open Sentences I then asked the students to look at Nick's sentence on the board. I said, "Let's try to write it as an open sentence using mathematical symbols. Let's use a variable for the number Nick picked. Who has an idea about how to do this?"

Diego said, "You could draw a box for the number. Then you write a multiplication sign and then a two and then a plus sign and a seven. Next put equals and the number fifty-seven." I wrote on the board:

$$\square \times 2 + 7 = 57$$

"You should put in parentheses," Lisa added. I added them:

$$(\square \times 2) + 7 = 57$$

"How could we write an open sentence for Carmen's?" I then asked.

Steve said, "Write parentheses, then box times two, then parentheses, then plus seven, and then an equals sign and forty-three." I recorded:

$$(\square \times 2) + 7 = 43$$

We did the same for Steve's, Lucy's, and Joshua's equations.

An Individual Assignment I then gave the class an assignment so that I could check their understanding. I wrote on the board:

Number multiplied by three, add eleven, results in fifty-six

I then gave directions, writing them on the board as I did so:

Write an open sentence using mathematical symbols.
Figure out the number that makes the open sentence true, and explain how you got it.

I circulated and observed the students. Few had any problem with this assignment. As I observed, I noticed that Jazmin had quickly written a correct open sentence and had started to figure out the answer. She subtracted eleven from fifty-six to get a difference of forty-five. Then she wrote a division problem—*45 ÷ 3*—but she didn't know how to proceed. I asked her what she thought she needed to do.

Jazmin explained, "I need to put forty-five things in three groups. I wonder if I could make three circles and then put tally marks in the circles one at a time until I have used up forty-five tallies."

"Try it and see what happens," I suggested. "I'll check back with you in a few minutes to see what you find out." When I checked back with Jazmin a little later, she had figured out that forty-five could be divided into three groups of fifteen. This information was enough for her to successfully complete the task. (See Figure 8–1.)

I noticed that Pablo seemed to be very intent on this assignment and I walked over to see what he was doing. He had figured out the answer using the "undoing" method, but then he wanted to also use the guess-and-check method. (See Figure 8–2.)

FIGURE 8–1 Jazmin drew three circles and used tally marks to figure out 45 ÷ 3.

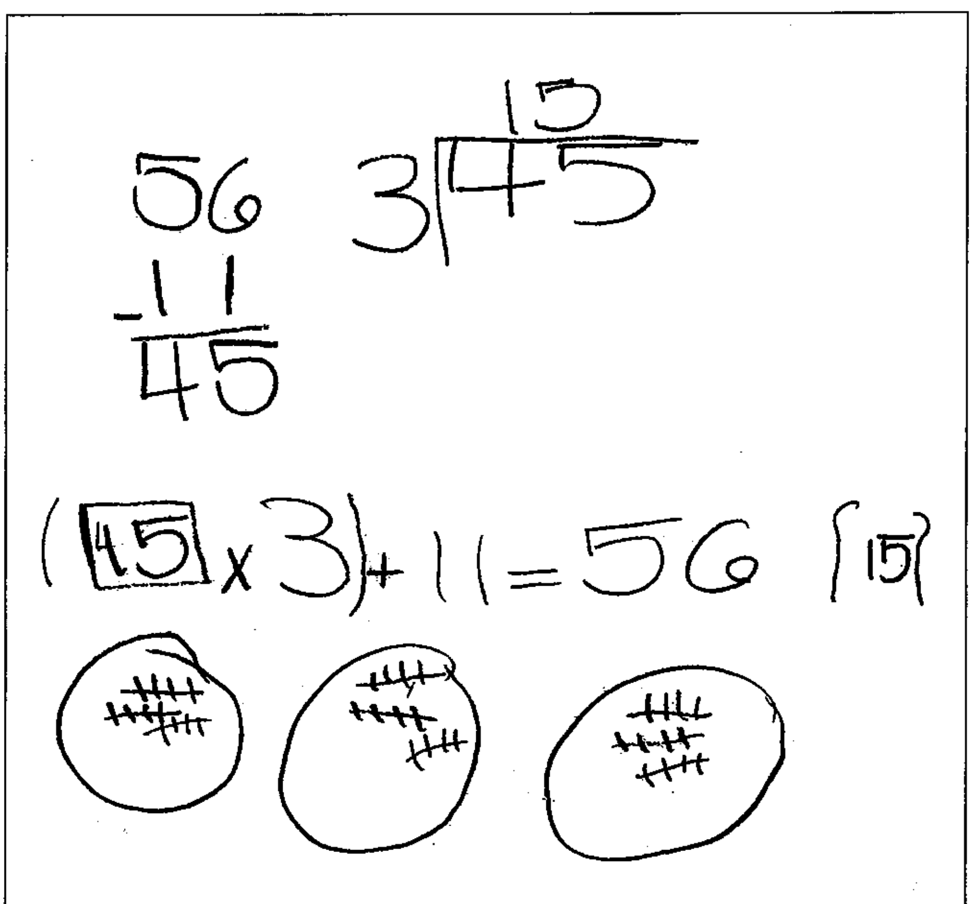


FIGURE 8-2 Pablo used the "undoing" method first, then the guess-and-check method.

$\square \times 3 + 11 = 56$

$\boxed{15} \times 3 + 11 = 56$

First I got 56 and subtracted 11 and it equaled 45 then I divided 45 by 3 and it equaled 15. The reason I did that was because it was like going back if you look back you can see insted of adding 11 I subtracted 11 and insted of multipling 3 I divided 3. Another reason I did that was because I wasn't adding up to fifty-six I was go down from fifty-six that why I subtracted and divided insted of adding and multipling.

I'm going to tell you another way to figure out an open problam. The way is first you guess a number under the number you have to get to then you multiply what you have to and add what you have to then you will probly not get the number you want so if it is to high of a number make the number you multipled and added, smaller if it is to small of a number make it higher. This could take a long time or a short time. Excample.

guess: 20

$\square \times 3 + 11 = 56$
$\square = 15$

20 x 3 = ~~60~~ It is already high so I'm going to make
19 x 3 = 57 it smaller.
18 x 3 = 54 It is already to high.
17 x 3 = 51, 51 + 11 = 65 It is to high.
16 x 3 = 48, 48 + 11 = 59 It is to high. 62 — It is to high.
15 x 3 = 45, 45 + 11 = 56 It is the right answer.

As students finished, I checked over their work, sometimes making corrections or suggestions. Figures 8–3 through 8–5 show how some other students solved this problem.

FIGURE 8–3 Kenny's work is typical of how the students used the guess-and-check method to solve the problem.

$\square \times 3 + 11 = 56$ 15

I got my anucer by picking a number and trying it and when I did 15 it worked I did try some other numbers that didn't work like 17 and 16 but they were too big.

FIGURE 8–4 Rayno wrote equations to explain how she used the "undoing" method.

$(\square \times 3) + 11 = 56$

$(\boxed{15} \times 3) + 11 = 56$

Explaination

$(56 - 11) \div 3 = \boxed{15}$

$(56 - 11 = \boxed{45}) \div 3 = 15$

$45 \div 3 = \boxed{15}$

$15 \times 3 = 45$

FIGURE 8–5 Gabe showed his calculations and wrote an explanation.

$$3 \times \square = \Delta \qquad \begin{array}{r} 15 \\ \times\ 3 \\ \hline ? \end{array} = \begin{array}{r} 15 \\ \times\ 3 \\ \hline 45 \\ +11 \\ \hline 56 \end{array}$$

$$\begin{array}{r} \Delta \\ +11 \\ \hline 56 \end{array}$$

$$3 \times \square = 45$$

$$\square = 15 \qquad \begin{array}{r} 45 \\ +11 \\ \hline 56 \end{array}$$

$$3 \times \Delta = \square$$

$$\begin{array}{r} \square \\ +11 \\ \hline 56 \end{array}$$

I found out you have 45 right before you add 11 So I did 3 X 15 which equals 45 because 2 X 15 equals 30 + 15 = 45 and 45 plus 11 = 56.

Extensions

1. Repeat the activity on other days, presenting other directions that involve the students with different numbers and different operations. Also, if the challenge would be appropriate for your students, you could have them pick a number between zero and fifty. Following are directions for possible follow-up experiences:

Number multiplied by three, minus seven, results in ______ $(\square \times 3) - 7$
Number plus one, then multiplied by five, results in ______ $(\square + 1) \times 5$
Number plus ten, then divided by two, results in ______ $(\square + 10) \div 2$

2. Have students make up their own examples. Give the following directions:

Think of an example with two operations and write it in words.
Write an open sentence to describe your example.

Then use the students' problems to provide the class with additional experiences. Either have a student read his or her example for others to write the equation, or have a student write the equation on the board and ask the others to give the words that go with it.